Projected Forward LIBOR and collateral currency

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Sharing Thoughts

SUMMARY

The purpose of this document is to provide a possible foundation that the projected forward LIBOR is independent of the collateral currency, provided that the "spread of the spread" is uncorrelated to the forward LIBOR.

Result: The projected forward of the currency i benchmark interest rate index LIBOR $L^{(i)}(T)$ under collateral currency k is independent of currency k if and only if the variables $e^{\int_{t}^{T}-y^{(i,k)}(s)ds}$ and $L^{(i)}(T)$ are uncorrelated. In this case, the projected forward LIBOR is given by the usual expression $E^{T^{i}}\left[L^{(i)}(T)\right]$.

Details

In the market there is a convention that the projected forward LIBOR is independent of the collateral currency. This document attempts to provide a theoretical justification of it and lay out the necessary and sufficient condition.

In [MFAT]: Derivative pricing formula when the payoff is in currency i while collateral is in currency k is given by $h_i^{(i)} = E^{\mathcal{Q}^i} \left[e^{-\int_t^T r^{(i)}(s)ds} e^{\int_t^T y^{(k)}(s)ds} h^{(i)}(T) \right]$, where $r^{(i)}(t)$ is the risk-free continuous compounding zero rate for currency i at time t, $c^{(k)}(t)$ be the continuous compounding collateral return rate for at time t, and $y^{(k)}(t) = r^{(k)}(t) - c^{(k)}(t)$, $h^{(i)}$ is the payoff.

That is
$$h_t^{(i)} = D^{(i)}(t,T)E^{T^i}\left[e^{\int_t^T - y^{(i,k)}(s)ds}h^{(i)}(T)\right]$$
 (see equation 5 of [MFAT]) where $D^{(i)}(t,T) = E^{Q^i}\left[e^{-\int_t^T e^{(i)}(s)ds}\right]$ and $y^{(i,k)}(t) = y^{(i)}(t) - y^{(k)}(t)$

Consider we have a discount curve that is bootstrapped to discount cash flows of currency i when collateral is in currency k. We can define $e^{-R^{(i,k)}(t,T)(T-t)} = DF(t,T) = D^{(i)}(t,T)E^{T^i}\left[e^{\int_t^T-y^{(i,k)}(s)ds}\right]$ where $R^{(i,k)}$ is the resulting discount zero curve that we already bootstrapped.

Consider now a payoff of the currency i benchmark interest rate index LIBOR $L^{(i)}(T)$ at time T in currency i when collateral is in currency k.

According to the formula, we will have $h_t^{(i)} = D^{(i)}(t,T)E^{T^i}\left[e^{\int_t^T - y^{(i,k)}(s)ds}L^{(i)}(T)\right]$. From the way that LIBOR

projected forward is used for pricing, this expression should be the same as $e^{-R^{(i,k)}(t,T)(T-t)}$ $PL^{(i)}(T)$ where PL stands for projected LIBOR. Equating the two, we get

$$PL^{(i)}(T) = e^{R^{(i,k)}(t,T)(T-t)}D^{(i)}(t,T)E^{T^{i}} \left[e^{\int_{t}^{T} - y^{(i,k)}(s)ds} L^{(i)}(T) \right]$$

$$= \frac{D^{(i)}(t,T)E^{T^{i}} \left[e^{\int_{t}^{T} - y^{(i,k)}(s)ds} L^{(i)}(T) \right]}{D^{(i)}(t,T)E^{T^{i}} \left[e^{\int_{t}^{T} - y^{(i,k)}(s)ds} \right]}$$

$$= \frac{E^{T^{i}} \left[e^{\int_{t}^{T} - y^{(i,k)}(s)ds} L^{(i)}(T) \right]}{E^{T^{i}} \left[e^{\int_{t}^{T} - y^{(i,k)}(s)ds} \right]}$$

This means, the necessary and sufficient condition for $PL^{(i)}(T) = E^{T^i}[L^{(i)}(T)]$, the projected LIBOR of the above expression while k = i, is to have

$$E^{T^{i}}\left[e^{\int_{t}^{T}-y^{(i,k)}(s)ds}\right]E^{T^{i}}\left[L^{(i)}\left(T\right)\right]=E^{T^{i}}\left[e^{\int_{t}^{T}-y^{(i,k)}(s)ds}L^{(i)}\left(T\right)\right]. \text{ In other words, the variables } e^{\int_{t}^{T}-y^{(i,k)}(s)ds} \text{ and } L^{(i)}\left(T\right) \text{ are } t^{T^{i}}\left[e^{\int_{t}^{T}-y^{(i,k)}(s)ds}L^{(i)}\left(T\right)\right]$$

uncorrelated (in the T^i forward measure)

This seems plausible since $y^{(i,k)}(t) = y^{(i)}(t) - y^{(k)}(t) = r^{(i)}(t) - c^{(i)}(t) - c^{(k)}(t) - c^{(k)}(t)$ is a spread of a spread, which may intuitively have nothing to do with currency i LIBOR.

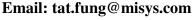
Reference

[MFAT] Masaaki Fujii, Akihiko Takahashi, Choice of collateral currency, RISK Jan 2011

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