Projected Forward LIBOR and collateral currency

Tat Sang Fung, PhD

Draft 1f:
Aug 2012

Sharing Thoughts

SUMMARY

The purpose of this document is to provide a possible foundation that the projected forward LIBOR is independent of the collateral currency, provided that the "spread of the spread" is uncorrelated to the forward LIBOR.

Result: The projected forward of the currency i benchmark interest rate index LIBOR $L^{(i)}(T)$ under collateral currency k is independent of currency k if and only if the variables $e^{\int_t^T -y^{(i,k)}(s)ds}$ and $L^{(i)}(T)$ are uncorrelated. In this case, the projected forward LIBOR is given by the usual expression $E^{T^i}\left[L^{(i)}(T)\right]$.

Details

In the market there is a convention that the projected forward LIBOR is independent of the collateral currency. This document attempts to provide a theoretical justification of it and lay out the necessary and sufficient condition.

In [MFAT]: Derivative pricing formula when the payoff is in currency i while collateral is in currency k is given by $h_t^{(i)} = E^{Q^i}\left[e^{-\int_t^T r^{(i)}(s)ds} e^{\int_t^T y^{(k)}(s)ds} h^{(i)}(T)\right]$, where $r^{(i)}(t)$ is the risk-free continuous compounding zero rate for currency i at time t, $c^{(k)}(t)$ be the continuous compounding collateral return rate for at time t. and $y^{(k)}(t) = r^{(k)}(t) - c^{(k)}(t)$, $h^{(i)}$ is the payoff.

That is $h_t^{(i)} = D^{(i)}(t,T) E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds} h^{(i)}(T)\right]$ (see equation 5 of [MFAT])

where $D^{(i)}(t,T) = E^{Q^i}\left[e^{-\int_t^T c^{(i)}(s)ds}\right]$ and $y^{(i,k)}(t) = y^{(i)}(t) - y^{(k)}(t)$

Consider we have a discount curve that is bootstrapped to discount cash flows of currency i when collateral is in currency k. We can define $e^{-R^{(i,k)}(t,T)(T-t)} = DF(t,T) = D^{(i)}(t,T) E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds}\right]$ where $R^{(i,k)}$ is the resulting discount zero curve that we already bootstrapped.

Consider now a payoff of the currency i benchmark interest rate index LIBOR $L^{(i)}(T)$ at time T in currency i when collateral is in currency k.

According to the formula, we will have $h_t^{(i)} = D^{(i)}(t,T)E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds}L^{(i)}(T)\right]$. From the way that LIBOR projected forward is used for pricing, this expression should be the same as $e^{-R^{(i,k)}(t,T)(T-t)}\ PL^{(i)}(T)$ where PL stands for projected LIBOR. Equating the two, we get

$$PL^{(i)}(T) = e^{R^{(i,k)}(t,T)(T-t)}D^{(i)}(t,T)E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds}L^{(i)}(T)\right]$$

$$= \frac{D^{(i)}(t,T)E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds}L^{(i)}(T)\right]}{D^{(i)}(t,T)E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds}\right]}$$

$$= \frac{E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds}L^{(i)}(T)\right]}{E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds}\right]}$$

This means, the necessary and sufficient condition for $PL^{(i)}(T) = E^{T^i}\left[L^{(i)}(T)\right]$, the projected LIBOR of the above expression while $k = i$, is to have $E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds}\right]E^{T^i}\left[L^{(i)}(T)\right] = E^{T^i}\left[e^{\int_t^T -y^{(i,k)}(s)ds}L^{(i)}(T)\right]$. In other words, the variables $e^{\int_t^T -y^{(i,k)}(s)ds}$ and $L^{(i)}(T)$ are uncorrelated (in the T^i forward measure)

This seems plausible since $y^{(i,k)}(t) = y^{(i)}(t) - y^{(k)}(t) = r^{(i)}(t) - c^{(i)}(t) - \left(r^{(k)}(t) - c^{(k)}(t)\right)$ is a spread of a spread, which may intuitively have nothing to do with currency i LIBOR.

Reference

[MFAT] Masaaki Fujii, Akihiko Takahashi, *Choice of collateral currency*, RISK Jan 2011

Contact

Tat Sang Fung, PhD
Head of Financial Engineering, Senior Manager, Product Management (Summit)
Global Financial Product Domain Leader, Product Management (Global)
Adjunct Assistant Professor, Mathematics Department, Columbia University

Email: tat.fung@misys.com
Academic Email: fts@math.columbia.edu
Linkedin: http://www.linkedin.com/pub/tat-sang-fung/6/367/192